POST GRADUATE DEPARTMENT OF MATHEMATICS
UNIVERSITY OF KASHMIR, SRINAGAR – 6

Syllabus for the course of study in
M.A/M.Sc. Mathematics (I to IV Semester)
For 2010-2013

***SEMESTER*-I**

(*Courses of Study for 2010 – 2013)*

Course No.	Title of the Course
MM-CP-101:---------------------	Abstract Algebra -I
MM-CP-102:---------------------	Real Analysis-I
MM-CP-103:----------------------	Complex Analysis-I
MM-CP-104:----------------------	Methods of Applied Mathematics-I
MM-CP-105:----------------------	Topology

SEMESESTER-II

(*Courses of Study for 2010 – 2013)*

Course No.	Title of the Course
MM-CP-201:-----------------------	Abstract Algebra-II
MM-CP-202: ---------------------	Real Analysis-II
MM-CP-203: ---------------------	Complex Analysis-II
MM-CP-204:---------------------	Methods of Applied Mathematics-II
MM-CP-205:---------------------	Functional Analysis - I

***SEMESTER*-III**

(*Courses of Study for the Year2011-2014)*

Core Courses

Course No.	Title of the Course
MM-CP-301: ---------------------	Ordinary Differential Equations
MM-CP-302: ----------------------	Functional Analysis-II

***SEMESTER*-IV**

(*Courses of Study for the Year 2011-2014)*

Core Courses

Course No.	Title of the Course
MM-CP-401:-----------------------	Partial Differential Equations
MM-CP-402:-----------------------	Differential Geometry

Optional Courses

Besides two core courses in 3rd and 4th Semesters as indicated above, three Optional papers out of the following will have to be chosen by a student in the 3rd Semester and the corresponding three optional papers in the 4th Semester keeping in view the suitability of the combinations.

SEMESTER: 3

(*Courses of Study for 2011-2014)*

Course No.	Title of the Course
MM-OP-303: ----------------------	Advanced topics in Topology & Modern Analysis
MM-CP-304:-----------------------	Abstract Measure Theory
MM-OP-305: -----------------------	Theory of Numbers-I
MM-OP-306:-----------------------	Advanced topics in Mathematical Modelling
MM-OP-307: -----------------------	Operations Research
MM-OP-308------------------------	Computer Programming
MM-OP-309: -----------------------	Advanced topics in Linear Algebra

SEMESTER: 4

(*Courses of Study for the Year 2011-2014)*

Course No.	Title of the Course
MM-OP-403------------------------ -	Advanced Topics in Functional Analysis
MM-OP-404 -------------------------	Advanced topics in the Analytic Theory of Polynomials
MM-CP-405 -------------------------	Theory of Numbers-II
MM-OP-406-------------------------	Advanced topics in Graph Theory
MM-OP-407-------------------------	Mathematical Statistics
MM-OP-408-------------------------	Wavelet Analysis
MM-OP-409-------------------------	Banach Algebras and Spectral Theory

Note:

The student shall have the option of choosing only those optional papers in the third & fourth semesters in which the facilities in terms of resource personnel & related infrastructure are available in the Department.

SEMESTER-I
ADVANCED ABSTRACT ALGEBRA-I

Course No. MM-CP-101 — ***Maximum Marks: 100***
Duration of Examination: 3 hrs — ***(a) External Exam: 80***
(b) Internal Exam: 20

Unit I

Definitions and Examples of Semi-groups and Monoids, Criteria for the semi-groups to be a group. Cyclic groups. Structure theorem for cyclic groups. Endomorphism, Automorphism, Inner Automorphism and Outer Automorphism, Center of a group, Cauchy's and Sylow's theorem for abelian groups. Applications of Sylow theory, Groups of order 2n, n as odd prime, groups of order p^2, pq, p^3. Permutation groups, Symmetric groups, Alternating groups, Simple groups, Simplicity of the Alternating group A_n for $n \geq 5$.

Unit II

Normalizer, conjugate classes, Class equation of a finite group and its applications, Cauchy's theorem, Sylow's theorem. Double cosets, Second and third parts of Sylow's theorem. Direct product of groups, Finite abelian groups, normal and subnormal series, Composition series. Jordan Holder theorem for finite groups. Zassenhaus Lemma, Schreir's Refinement theorem, Solvable groups and Nilpotent groups.

Unit III

Brief review of Rings, Integral domain, Ideals. The field of quotients of an Integral domain. Euclidean rings with examples such as $Z[\sqrt{-1}]$, $Z[\sqrt{2}]$, Principal ideal rings(PIR) Unique factorization domains(UFD), Relationships between Euclidean rings, P.I.R.'s and U.F.D. The Division algorithm for polynomials, Irreducible polynomials, Polynomials and the rational field, Primitive polynomials, Contract of a polynomials, Gauss Lemma, Integer monic polynomial, Eisenstein's irreducibility criterion, Polynomial rings and Commutative rings.

Unit IV

Canonical forms: Triangular form, Invariance, Invariant direct-sum decomposition, Primary decomposition, Nilpotent operators, Jordon canonical form, cyclic subspaces, Rational canonical form, Quotient spaces. Bilinear forms, Alternating Bilinear forms, Symmetric bilinear forms, quadratic forms, Law of inertia, Hermitian forms.

Recommended Books:

1. I.N.Heristein : Topics in Algebra.
2. K.S.Miller : Elements of Modern Abstract Algera.
3. Surjeet Singh and Qazi Zameer-ud-din: Modern Algebra, Vikas Publishing House Private Limited.
4. P.B,.Bhatacharaya and S.K.Jain : Basic Abstract Algebra.
5. J.B. Fragleigh : A First Course in Abstract Algebra.
6. J.A.Gallian : Contemporary Abstract Algebra.

REAL ANALYSIS-I

Course No. MM-CP-102 — *Maximum Marks: 100*
Duration of Examination: 3 hrs — *(a) External Exam: 80*
(b) Internal Exam: 20

Unit I

Infinite series: Carleman's theorem. Conditional and absolute convergence, multiplication of series, Merten's theorem, Riemann's rearrangement theorem.
Sequence and series of functions: Pointwise and uniform convergence, Cauchy criterion for uniform convergence, Weirstrauss M-test, uniform convergence and continuity, Riemann integration and differentiation, Weirstrass's Approximation Theorem, Example of a continuous nowhere differentiable function on R.

Unit II

Integration : Definition and existence of Riemann – Stieltjes integral , behavior of upper and lower sums under refinement, Necessary and sufficient conditions for RS-integrability of continuous and monotonic functions , Reduction of an RS-integral to a Riemann integral , Basic properties of RS-integrals, Differentiability of an indefinite integral of a continuous function, Fundamental theorem of calculus for Riemann integrals.

Unit III

Improper Integrals: Integration of unbounded functions with finite limits of integration. Comparison test for convergence of improper integrals, Cauchy's test, Infinite range of integration. Absolute convergence. Integrand as a product of functions. Abel's and Dirichlet's test, Elementary functions- a rigorous introduction.
Inequalities: Arithmetic-geometric means equality, Inequalities of Cauchy Schwartz, Jensen, Holder& Minkowski. Inequality on the product of arithmetic means of two sets of positive numbers**.**

Unit IV

Functions of several variables, directional derivative and continuity, total derivative, Matrix of a linear function, Jacobian matrix, chain rule, mean value theorem for differentiable functions. Sufficient conditions for differentiability and for the equality of mixed partials, Taylor's theorem for functions from R^n and R. Inverse and Implicit function theorems in R^n . Extremum problems for functions on R^n .Langrange's multipliers ,Multiple Riemann Integral and change of variable formula for multiple Riemann integrals**.**

<u>Recommended Books:</u>

1. R. Goldberg : Methods of Real Analysis
2. W.Rudin : Principles of Mathematical Analysis
3. J.M.Apostol : Mathematical Analysis
4. S.M.Shah and Saxena: Real Analysis
5. A.J.White :Real Analysis , An Introduction
6. L.Royden :Real Analysis

COMPLEX ANALYSIS-I

Course No. MM-CP-103 — ***Maximum Marks: 100***
Duration of Examination: 3 hrs — ***(a) External Exam: 80***
(b) Internal Exam: 20

Unit I

Review of C-R equations and analytic functions. Complex integration, Cauchy Goursat theorem, the index of a point w.r.t. of a closed curve. Cauchy's integral formula, higher order derivatives. Morera's theorem, Cauchy's inequality and Liounillis Theorem.

Unit II

Power Series Cauchy- Hadamard formula for the radius of convergence. Taylor's Theorem, Taylor Series. Expansion of an analytic function in a power series. Lourant series and isolated singularities, poles and essential singular points. Behavior at an essential singular point, Casorati-Weirstrauss Theorem.

UnitIII

Bilinear(Moebus) transformations. Their properties and classification. Fixed Points, Cross Ratio, Inverse points and Crictical Points. Conformal Mapping. Mappings of (i) upper half plane on to the unit disc, (ii) unit disc on to the unit disc, (iii) left half plane on to the unit disc and (iv) circle on to a circle. The Transformations $w = \sqrt{z}$, $w = z^2$ *and* $w = \frac{1}{2}\left(z + \frac{1}{z}\right)$.

UnitIV

Residues: Cauchy Residues Theorem, Evaluation of Integers by the method of Residue, Pasevals identity, Branches of many valued functions with special reference to arg.(Z), log (Z), Z^a. Infinite products, convergence and divergence of infinite products.

Recommended Books:

1. L.Ahlfors, Complex Analysis.
2. E.C.Titchmarsh , Theory of functions .
3. J.B.Conway ,Functions of a Complex Variable-1.
4. Richard Silverman, Complex Analysis.
5. H.A.Priestly, Introduction to complex Analysis.
6. Nihari Z. Conformal mappings.

METHODS OF APPLIED MATHEMATICS-I

Course No. MM-CP-104 | ***Maximum Marks: 100***
Duration of Examination: 3 hrs | ***(a) External Exam: 80***
(b) Internal Exam: 20

UNIT-I(Matrix Theory)

Eigen values and eigen vectors of a matrix and their determination.Similarity of matrices. Two similar matrices have the same eigen values. Algebraic and geometric multiplicity of a characteristic root. Necessary and sufficient condition for a square matrix of order n to be similar to a diagonal matrix. Orthogonal reduction of real matrices, Schur's theorem. Normal matrices, Necessary and sufficient condition for a square matrix to be unitarily similar to a diagonal matrix.

Quadratic forms: The Kroneckers and Lagranges reduction .Reduction by orthogonal transformation of real quadratic forms .Necessary and sufficient condition for a quadratic form to be positive definite. Rank , Index and signature of a quadratic form. If $A=[a_{ij}]$ is a positive definite matrix of order n , then $|A| \leq a_{11}a_{22}\cdots a_{nn}$. Gram matrices. The Gram matrix $B'B$ is always positive definite or positive semi-definite. Hadmard's inequality. If $B=[b_{ij}]$ is an arbitrary non- singular real square matrix of order n, then $|B| \leq \prod_{i=1}^{n}\left[\sum_{k=1}^{n} b_{ik}\right]$.

UNIT-II
(Numerical Analysis)

Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

(Theory of Probability)

UNIT- III

The probability set functions, its properties, probability density function, the distribution function and its properties. Mathematical Expectations, some special mathematical expectations, Inequalities of Makov, Chebyshev and Jensen.

UNIT-IV

Conditional probability, independent events, Baye's theorem, Distribution of two and more random variables, Marginal and conditional distributions, conditional means and variances, Correlation coefficient, stochastic independence and its various criteria.

Recommended Books:

1 Introduction to Matrix Analysis by Richard Bellman , McGraw Hill Book Company.
2 Introduction to Numerical Analysis by K.E. Atkinson
3 Hogg and Craig : An Introduction to the Mathematical Statistics

Suggested Readings:

1. Elementary Matrix Algebra by Franz E. Hohn, American Publishing company Pvt.ltd.
2. A Text Book of Matrices by Shanti Narayan, S. Chand and company Ltd.
3. Introduction to Methods of Numerical Analysis by S.S.Sastry.
4 Mood and Grayball : An Introduction to the Mathematical Statistics

TOPOLOGY

Course No. MM-CP-105 — ***Maximum Marks: 100***
Duration of Examination: 3 hrs — ***(a) External Exam: 80***
(b) Internal Exam: 20

Unit I

Review of countable and uncountable sets, Schroeder-Bernstein theorem, Axiom of Choice and its various equivalent forms, Definition and examples of metric spaces, Open and Closed sets, Nets in topological spaces, convergence of nets, completeness in metric spaces, Baire's Category theorem, and applications to the (1) Non-existence of a function which is continuous precisely at irrationals (ii) Impossibility of approximating the characteristic of rationals on [0, 1] by a sequence of continuous functions.

Unit II

Completion of a metric space ,Cantor's intersection theorem, with examples to demonstrate that each of the conditions in the theorem is essential, Uniformly continuous mappings with examples and counter examples, Extending Uniformity continuous maps, Banach's contraction principle with applications to the inverse function theorem in R.

Unit III

Topological spaces; Definition and examples, elementary properties, Kuratowski's axioms, continuous mappings and their characterizations ,pasting Lemma, convergence of nets and continuity in terms of nets, Bases and sub bases for a topology, Lower limit topology, concepts of first countability, second countability, separability and their relationships, counter examples and behavior under subspaces, product topology and weak topology, compactness and its various characterizations.

Unit IV

Heine-Borel theorem, Tychnoff's theorem, compactness, sequential compactness and total bounded ness in metric spaces. Lebesgue's covering lemma, continuous maps on a compact space. Separation axioms T_i (i=1,2,3,3 1/2,4) and their permanence properties, connectedness, local connectedness, their relationship and basic properties, Connected sets in R. Urysohn's lemma. Urysohn's metrization theorem. Tietize's extension theorem, one point compactification.

Recommended Books:

1. G.F.Simmons : Introduction to topology and Modern Analysis
2. J. Munkres : Topology
3. K.D. Joshi : Introduction to General topology
4. J.L.Kelley : General topology
5. Murdeshwar ; General topology
6. S.T. Hu : Introduction to General topology

SEMESTER: 2

ADVANCED ABSTRACT ALGEBRA—II

Course No. MM-CP-201 | ***Maximum Marks: 100***
Duration of Examination:2-1/2 hrs | ***(a) External Exam: 80***
(b) Internal Exam: 20

Unit I

Relation and Ordering, partially ordered sets, Lattices, properties of Lattices, Lattices as algebraic Systems, sub-lattices, direct product and homomorphism, Modular Lattices, complete Lattices, bounds of Lattices, Distributive Lattice, Complemented Lattices.
Introduction, definition and important properties of Boolean Algebra, Sub Boolean algebra, Direct product and homomorphism, Join-irreducible, Meet-irreducible, Atoms, Stone's representation theorem. Boolean expressions and their equivalence, Free Boolean algebra, Values of Boolean expression, representation of Boolean function, Karnaugh maps, Minimization of Boolean function.

Unit-II

Modules, Sub-modules, Quotient Modules, Homomorphism and Isomorphism theorem. Cyclic Modules, Simple Modules, Semi-Simple Modules, Schuler's Lemma, Free Modules. Ascending chain condition and Maximum condition, and their equivalence. Descending chain condition and Minimum condition, and their equivalence. direct sums of modules. Finitely generated modules.

Unit-III

Fields: Prime fields and their structure, Extensions of fields, Algebraic numbers and Algebraic extensions of a field, Roots of polynomials, Remainder and Factor theorems, Splitting field of a polynomial, Existence and uniqueness of splitting fields of polynomials , Simple extension of a field.

Unit IV

Separable and In-separable extensions, The primitive element theorem, Finite fields, Perfect fields, The elements of Galois theory. Automorphisms of fields, Normal extensions, Fundamental theorem of Galois theory, Construction with straight edge and compass, R^n is a field iff $n = 1, 2$.

Recommended Books:

7. I.N.Heristein : Topics in Algebra.
8. K.S.Miller : Elements of Modern Abstract Algera.
9. Surjeet Singh and Qazi Zameer-ud-din: Modern Algebra, Vikas Publishers Pvt. Limited.
10. P.B,.Bhatacharaya and S.K.Jain : Basic Abstract Algebra.
11. J.B. Fragleigh : A First Course in Abstract Algebra.

12.J.A.Gallian : Contemporary Abstract Algebra.

REAL ANALYSIS-II

Course No. MM-CP-202 ***Maximum Marks: 100***
Duration of Examination:3 hrs ***(a) External Exam: 80***
(b) Internal Exam: 202

Unit I

Measure theory: Definition of outer measure and its basic properties, Outer measure of an interval as its length. Countable additivity of the outer measure, Borel measurable sets and Lebesgue measurability of Borel sets, Cantor set, existence of non- measurable sets and of measurable sets which are not Borel, Outer measure of monotonic sequences of sets.

UNIT-II

Measurable functions and their characterization. Algebra of measurable functions, Stienhauss theorem on sets of positive measure, Ostrovisk's theorem on measurable solution of $f(x+y)= f(x) + f(y)$,x,y ЄR. Convergence a.e., convergence in measure and almost uniform convergenc, their relationship on sets of finite measure, Egoroff's theorem.

UNIT-III

Lebesgue integral of a bounded function. Equivalence of L-integrability and measurability for bounded functions, Riemann integral as a Lebesgue integral, Basic properties of Lebesgue –integral of a bounded function. Fundamental theorem of calculus for bounded derivatives. Necessary and sufficient condition for Riemann integrability on [a, b]. L- integral of non- negative measurable functions and their basic properties. Fatou's lemma and monotone convergence theorem. L–integral of an arbitrary measurable function and basic properties. Dominated convergence theorem and its applications.

UNIT-IV

Absolute continuity and bounded variation , their relationships and counter examples. Indefinite integral of a L-integrable functions and its absolute continuity. Necessary and sufficient condition for bounded variation. Vitali's covering lemma and a.e. differentiability of a monotone function f and $\int f^{/} \leq f(b)-f(a)$.

Recommended Books:

1. Royden, L. :Real Analysis (PHI)
2. Goldberg , R. : Methods of Real Analysis
3. Barra, De. G. : Measure theory and Integration (Narosa)
4. Rana ,I.K. : An Introduction to Measure and Integration.
5. Rudin, W. Principles of Mathematical Analysis.
6. Chae, Lebesgue Integration.
7. T.M.Apostol : Mathematical Analysis

8. S.M.Shah and Saxena : Real Analysis

COMPLEX ANALYSIS-II

Course No. MM-CP-203 ***Maximum Marks: 100***
Duration of Examination: 3 hrs ***(a) External Exam: 80***
(b) Internal Exam: 20

Unit I

Maximum Modulus Principle, Schwarz Lemma and its generalization, Meromorphic function, Argument Principle, Rouche's theorem with application, Inverse function Theorem, Poisson integral formula for a circle and half plane, Poisson Jenson formula, Carleman's theorem, Hadmard three-circle theorem and the theorem of Borel and Caratheodory**.**

Unit II

Principle of analytic continuation, uniqueness of direct analytical continuations and uniqueness of analytic continuation along a curve. Power series method of analytic continuation, Functions with natural boundaries and related examples. Shewartz reflection principle, functions with +ive real part. Monodromy theorem and its applications.

Unit III

Space of analytic functions, Hurwitz's theorem, Montel's theorem, Riemann Mapping theorem, Weistrass factorization theorem, Gamma function and its properties, Riemann Zeta function, Reimann's functional equation. Harmonic functions on a disc, Harnack's inequlity and theorem, Drichlet's problem, Green's functions.

Unit IV

Canonical products, order of an entire functions, Exponential convergence, Borel theorem, Hadmards factorization theorem, the range of analytic function, Bloch's theorem, Schottkys theorems, the little Picard's theorem, Landau's theorem, Montel Caratheadory theorem and the Great Picard theorem. Univalent function. Bieberbach's conjecture (statement only) and the 1/4 – theorem.

<u>Recommended Books:</u>

1. L.Ahlfors: Complex Analysis
2. E.C. Titchmarsh : Theory of Functions
3. J.B.Conway : Functions of a complex variable –I
4. Richard's Silverman : Complex Analysis
5. A.I.Markushevish :Theory of Functions of a Complex variable
6. Nihari Z. : Conformal Mapping.
7. H.A. Priestly : Introduction to Complex Analysis.
8. S.Lang : Complex Analysis.
9. E.Hille : Analytic Function Theory (2- vol).
10. Liang –Shin Hahn, Bernard Epstein : Classical Complex Analysis.
11. D.Sarason: Complex Function Theory
12. W.H.J.Fuchs :Topics in the Theory of Functions on one Complex Variable.

METHODS OF APPLIED MATHEMATICS-II

Course No.MM-CP-204 — ***Maximum Marks: 100***

Duration of Examination: 3 hrs — ***(a) External Exam: 80***

(b) Internal Exam: 204

Unit-I

Graphs, Basic terminology, Incidence and Degree, Isomorphism, Sub graphs, adjacency matrix, Walks, Paths, Cycles, Connected graphs, Components, Eulerian graphs, Euler's theorem, Konigsberg Bridge Problem, Unicursal graphs, Operations on graphs, connected graphs and circuits, Hamiltonian paths and cycles, Dirac's theorem, Degree sequences. Planar graphs, Kuratowski's two graphs, Embedding on a sphere, Euler's formula.

Unit-II

Trees, properties of trees, Pendant vertices in trees, Degree sequences in trees, Necessary and sufficient conditions for a sequence to be a degree sequence of a tree, Distance and Centers in a tree, spanning tree of a graph, The minimum spanning tree problem, Rooted and Binary trees, Cayley's theorem on the number of trees on a given set of vertices, Fundamental cycles, Generation of trees, Ramsey's theorem and Ramsey numbers.

Unit-III

Introduction to Mathematical Modelling, Types of Modelling, Mathematical formulation of a problem, Solution and Interpretation of a Model. Modelling Motion of a Simple Pendulum, Simple Harmonic Motion, Escape Velocity, Kepler's Planetary Laws, Single Species Population Models, Exponential Growth Model and Logistic Growth Model.

Unit-IV

Modelling Blood Flow and Oxygen transfer in Human Body, Viscosity, Poiseulla Law and their Mathematical Formulation, Constituents of Blood, Blood Circulation in Heart. Fick's Law of Diffusion and Fick's Perfusion Principle, Diffusion in Biological Systems, Olfactory Communication in Animals.

<u>Recommended Books:</u>

1. F. Harary, Graph Theory, Addison-Wesley.
2. Narsingh Deo : Graph Theory with Applications to Engineering and Computer Sciences, Printice Hall, India Ltd.
3. D.B. West Introduction to Graph Theory prentice, Hall, India.
4. J. Clark and D.A Holton: A First book at Graph Theory, World Scientific
5. O. Ore: Theory of Grpahs, AMS.
6. J.Matousek and J.Nesetril, Invitation to Discrete Mathematics, Oxford University Press, 2009.

7. J.D. Murray Mathematical Biology (An Introduction, Vol. I and II), Springer Verlag.
8. J.N. Kapur, Mathematical Model in Biology and Medicines.
9. S. I. Rubinow, Introduction to Mathematical Biology, John Wiley and Sons, 1975.
10. M. R. Cullen, Linear Models in Biology, Ellis Horwood Ltd.
11. J. N. Mazumdar, Biofluid Dynamics, World Scientific, Singapore.

FUNCTIONAL ANALYSIS

Course No. MM-CP-205 ***Maximum Marks: 100***
Duration of Examination: 3 hrs ***(a) External Exam: 80***
(b) Internal Exam: 20

BANACH SPACE:
Unit I
Banach Spaces: Definition and examples, subspaces, quotient spaces, Continuous Linear Operators and their Characterization, Completeness of the space L(X,Y) of bounded linear operators (and its converse), incompleteness of C[a, b], under the integral norm, Finite dimensional Banach spaces, Equivalence of norms on finite dimensional space and its consequences, Dual of a normed linear space, Hahn Banach theorem (extension form) and its applications, Complemented subspaces, Duals of l_p^n, C_o, l_p ($p\geq1$), C[a, b].

Unit II
Uniform boundedness Principle and weak boundedness, Dimension of an ∞-dimensional Banach space, Conjugate of a continuous linear operator and its properties, Banach-Steinhauss theorem, open Mapping and closed graph theorems, counterexamples to Banach-Steinhauss, open mapping theorem and closed graph theorems for incomplete domain and range spaces, separable Banach spaces and the separability of some concrete Banach spaces (C_o, C[0,1], l_p, $p\geq1$), Reflexive Banach Spaces, closed subspace and the dual of a reflexive Banach space, Examples of reflexive and non-reflexive Banach spaces.

and examples, **HILBERT SPACE**:
Unit III
Hilbert spaces: Definition and examples, Cauchy's Schwartz inequality, Parallogram law, orthonormal (o.n) systems, Bessel's inequality and Parseval's Identity for complete orthonormal systems, Riesz-Fischer theorem, Gram Schmidt process, o.n basis in separable Hilbert spaces.
Fourier Series: Fourier series with respect to an o.n. base in Hilbert space, Applications to classical Fourier analysis, Examples of special o.n. bases in $L_2[-\pi,\pi]$. Convergence of Fourier series: Fejer's theorem on (C,1) convergence of Fourier series of a continuous function on $(-\pi,\pi)$, Existence of a continuous function with a divergent Fourier series at a point, Riemann- Lebesgue Lemma, Convergence of Fourier series of

a function which is continuous and has left and right hand derivatives, Term by term integration of Fourier series, Uniform convergence of a Fourier series

UNIT-IV

Projection theorem, Riesz Representation theorem. Counterexample to the Projection theorem and Riesz Representation theorem for incomplete spaces. Hilbert property of the dual of a Hilbert space and counter examples of incomplete inner product spaces, Reflexivity of Hilbert space, Adjoint of a Hilbert space operator, weak convergence and Bolzano-Weirstrass property in Hilbert Spaces. Normal and Unitary operators, Finite dimensional spectral theorem for normal operators**.**

Recommended Books:

1. B.V.Limaya: Funtional Analysis.
2. C.Goffman G. Pedrick: A First Course in Functional Analysis.
3. L.A. Lusternick & V.J. Sobolov. : Elements of Functional Analysis
4. J.B. Conway : A Course in Functional Analysis.

***SEMESTER:*3**

ORDINARY DIFFERENTIAL EQUATIONS

Course No. MM-CP-301   ***Maximum Marks: 100***
Duration of Examination: 3 hrs   ***(a) External Exam: 80***
(nternal Exam: 20

Unit I

First order ODE, Singular solutions, p-discriminate and c-discriminate, Initial value problems of first order ODE, General theory of Homogeneous and Non-homogeneous linear ODE, Picard's theorem on the existence and uniqueness of solutions to an initial value problem, Factorization of Operator. Method of variation of parameters.

Unit II

Solution in Series: (i) Roots of an Indicial equation, un-equal and differing by a quantity not an integer. (ii) Roots of an Indicial equation, which are equal. (iii) Roots of an Indicial equation differing by an integer making a coefficient infinite. (iv) Roots of an Indicial equation differing by an integer making a coefficient indeterminate.
Simultaneous equation dx/P = dx/Q = dz/R and its solutions by use of multipliers and a second integral found by the help of first. Total differential equations Pdx + Qdy +Rdz = 0. Necessary and sufficient condition that an equation may be integrable. Geometric interpretation of the Pdx + Qdy + Rdz =0.

Unit III

Existence of Solutions, Initial value problem, Ascoli- lemma, Cauchy Piano existence theorem, Uniqueness of solutions with examples, Lipschitz condition and Gronwall inequality, Method of successive approximation, Picard-Lindlof theorem, Continuation of solutions, System of Differential equations, Dependence of solutions on initial conditions and parameters.

Unit IV

Maximal and Minimal solutions of the system of Ordinary Differential equations, Cartheodary theorem, Linear differential equations, Linear Homogeneous equations, Linear system with constant coefficients, Linear systems with periodic coefficients, Fundamental matrix and its properties, Non-homogeneous linear systems, Variation of constant formula. Wronskian and its properties.

<u>Recommended Books:</u>

1. H.T.H. Piaggio, Differential Equations, CBS Publishers and Distributors, New Delhi
2. P.Hartmen : Ordinary Differential Equations
3. W.T.Reid : Ordinary Differential Equations
4. E.A.Coddington and N.Levinson :Theory of Ordinary Differential Equations.
5. D. Somasundaram, Ordinary Differential Equations, Narosa Publishers, New Delhi.

FUNCTIONAL ANALYSIS-II

Course No. MM-CP-302
Duration of Examination: 3 hrs

Maximum Marks: 100
(a) External Exam: 80
(b) Internal Exam: 20

Unit I

Relationship between analytic and geometric forms of Hahn-Banach Theorem, Applications of Hahn-Banach Theorem: Banach limits, Markov-Kakutani theorem for a commuting family of maps, Complemented subspaces of Banach spaces, Complentability of dual of a Banach space in its bidual, uncomplementability of co, Dual of Subspace, Quotient space of a normed space.

Unit II

Banach's closed range theorem, injective and surjective bounded linear mappings between Banach spaces ℓ_∞ and C[0,1] as universal separable Banach spaces, l_1 as a quotient universal separable Banach space, Weak and weak* topologies on a Banach space, Goldstine's theorem, Banach-Alaoglu theorem and its simple consequences.

Unit III

Reflexivity of Banach spaces and weak compactness, Completeness of Lp[a,b]. Duals of ℓ_∞, C(X) and Lp spaces, Banach Stone Theorem, Applications of fundamental theorems to Radon-Nikodym Theorem, Laplace transform.

Unit IV

Extreme points, Krein-Milman theorem and its simple consequences, Mazur-Ulam theorem on isometries between real normed spaces, Muntz theorem on C[a,b] and L2 [a,b]. Bases in Banach spaces, Schauder basis for C[0,1].

Recommended Books:

1. Ballobas, B;Lineart Analysis(Camb. Univ.Pres)
2. Goffman, C and Pedrick ,G; A first course in functional Analysis (Prentice Hall.)
3. Beauzamy, B; Indroduction to Banach Spaces and their geometry (North Holland).
4. Rudin, W; Functional Analysis (Tata McGrawHill).

ADVANCED TOPICS IN TOPOLOGY AND MODERN ANALYSIS

Course No. MM-CP-303 ***Maximum Marks: 100***
Duration of Examination: 3 hrs ***(a) External Exam: 80***
(b) Internal Exam: 20

Unit I

Uniform spaces. Definition and examples, uniform topology, and metrizability complete regularity of uniform spaces, pre-compactness and compactness in uniform spaces, uniform continuity.

Unit II

Uniform continuity, uniform continuous maps on compact spaces Cauchy convergence and completeness in uniform spaces, initial uniformity, simple applications to function spaces, Arzela- Ascoli theorem**.**

Unit III

Abstract Harmonic Analysis, Definition of a topological group and its basic properties. Subgroups and quotient groups. Product groups and projective limits. Properties of topological groups involving connectedness. Invariant metrics and Kakutani theorem, Structure theory for compact and locally compact Abelian groups.

Unit IV

Some special theory for compact and locally compact Abelian groups, Haar integral and Haar measure, invariant means defined for all bounded functions, convolution of functions and measures. Elements of representation theoy, Unitary representations of locally compact groups.

Recommended Books:

1. I.M. James Uniform Spaces, Springer Verlag.
2. K.D. Joshi, Introduction to General Topology.
3. S.K.Berberian, Lectures on Operator Theory and Functional Analysis, Springer Verlag.
4. G.B. Folland, Real Analysis, John Wiley.

Suggested Readings:

1. G. Murdeshwar, General Topology,
2. E. Hewitt & K.A Ross, Abstract harmonic Analysis-I, Springer Verlag.

ABSTRACT MEASURE THEORY

Course No. MM-CP-304
Duration of Examination: 3 hrs

Maximum Marks: 100
(a) External Exam: 80
(b) Internal Exam: 20

Unit-I

Semiring , algebra and σ - algebra of sets, Borel sets, measures on semirings, outer measure associated with a set function and basic properties, measurable sets associated with an outer measure as a σ - algebra , construction of the Lebesgue measure on R^n .

Unit-II

For f Є L_1 [a, b], $F^{/}$= f a.e. on [a, b]. If f is absolutely continuous on (a, b) with f(x)=0 a.e, then f = constant. Characterization of an absolutely continuous function as an indefinite Lebesgue integral. Non-Lebesgue integrability of f where f (x) = $x^2\sin(1/ x^2)$, f (0) =0 on [0, 1]. Fundamental theorem of calculus for the Lebesgue integral. A brief introduction to L_p spaces. Holder's and Minkowki's inequalities.

Unit-III

Improper Riemann integral as a Lebesgue integral, calculation of some improper Riemann integrable functions, space of Lebesgue integrable functions as completion of Riemann integrable functions on [a,b], change of variables formula and simple consequences, Riemann Lebesgue lemma.

Unit-IV

Product measures and iterated integrals, example of non-integrable functions whose iterated integrals exist (and are equal), Fubini theorem, expressing a double integral as an iterated integral, Tonelli-Hobson theorem as a converse to Fubini theorem, differentiation under the integral sign.

Recommended Books:

1.C.D.Aliprantis and O.Burkinshaw, Principles of Real Analysis
2.Goldberg , R. : Methods of Real Analysis
3.T.M.Apostol : Mathematical Analysis

Suggested Readings:

1.Royden, L: Real Analysis (PHI)
2.Chae, S.B. Lebesgue Integration(Springer Verlag).
3.Rudin, W. Principles of Mathematicals Analysis(McGraw Hill).
4.Barra ,De. G. : Measure theory and Integration (Narosa)
5.Rana ,I.K. : An Introduction to Measure and Integration, Narosa Publications.

THEORY OF NUMBERS-I

Course No. MM-CP-305 — ***Maximum Marks: 100***
Duration of Examination: 3 hrs — ***(a) External Exam: 80***
(b) Internal Exam: 20

Unit I

Divisibility, the division algorithm and its uniqueness, Greatest common divisor and its properties. The Euclidean algorithm, Prime numbers. Euclid's first theorem, Fundamental Theorem of Arithmetic, Divisor of n, Radix-representation Linear Diophantine equations. Necessary and sufficient condition for solvability of linear Diophantine equations, Positive solutions.

Unit II

Sequence of primes, Euclid's Second theorem, Infinitude of primes of the form 4n+3 and of the form 6n+5. No polynomial f(x) with integral coefficients can represent primes for all integral values of x or for all sufficiently large x. Fermat Numbers and their properties. Fermat Numbers are relatively prime. There are arbitrary large gaps in the sequence of primes. Congruences, Complete Residue System (CRS), Reduced Residue System (RRS) and their properties. Fermat and Euler's theorems with applications.

Unit III

Euler's ϕ-function, ϕ (mn) = ϕ (m) ϕ (n) where (m, n) =1, $\sum_{d/m}\phi(d)=n$ and $\phi(m)=m\prod_{p}\left(1-\frac{1}{p}\right)$ for m>1. Wilson's theorem and its application to the solution the congruence of $x^2 \equiv -1 \pmod p$, Solutions of linear Congruence's. The necessary and sufficient condition for the solution of $a_1x_1+a_2x_2+\ldots+a_nx_n \equiv c \pmod m$. Chinese Remainder Theorm.Congruences of higher degree F(x) ≡ 0 (mod m), where F(x) is a Polynomials. Congruence's with prime power, Congruences with prime modulus and related results. Lagrange's theorem, viz , the polynomial congruence $F(x) \equiv 0 \pmod p$ of degree n has at most n roots.

Unit IV

Factor theorem and its generalization. Polynomial congruences $F(x_1,x_2 \dots x_n) \equiv 0 \pmod p$ in several variables. Equivalence of polynomials. Theorem on the number of solutions of congruences: Chevalley's theorem, Warning's theorem. Quadratic forms over a field of characteristic $\neq 2$ Equivalence of Quadratic forms. Witt's theorem .Representation of Field Elements. Hermite's theorem on the minima of a positive definite quadratic form and its application to the sum of two squares.

Recommended Books:

1. Topics in number theory by W. J . Leveque, Vol. I and II Addition Wesley Publishing Company, INC.
2. An introduction of the Theory of numbers by I. Niven and H.S Zuckerman.
3. Number Theory by Boevich and Shaferivich, I.R, Academic Press.

Suggested Readings:

1. Analytic Number Theory by T.M Apostal, Springer Verlag.
2. An introduction to the theory of Numbers by G.H Hardy and Wright.
3. A course in Arithmetic, by J.P. Serre, GTM Vol. springer Verlag 1973.
4. An elementary Number theory of E. Landau.

MATHEMATICAL BIOLOGY

Course No. MM-CP-306 ***Maximum Marks:100***
Duration of Examination: 3 hrs ***(a) External Exam: 80***
(b) Internal Exam: 20

Unit-I

Two species Population Models, Types of Interactions between two Species, Prey-Predator Model, Lotka-Voltera Systems and its Geometrical Interpretation, Competition Models, Mutualism and Symbiosis, Stability Analysis of Prey-Predator Model.

Unit-II

Epidemic Models and Dynamics of Infectious Diseases: Simple Epidemic Models; SIS, SIR and SRS Epidemic Models. Modelling Venereal Diseases, Modelling Transmission Dynamics of HIV.

Unit-III

Cell Growth, Exponential Growth or Decay, Determination of Growth or Decay Rates, The method of Least Squares, Nutrient Uptake by a Cell, Growth of Microbial Colony and Growth of Chemostat.

Enzyme kinetics, The Michaelis-Menten Theory, Early time behaviour of Enzymatic reactions, Cooperative properties of Enzymes, Allosteric Enzymes, Haemoglobin,

Unit-IV

Introduction to compartment models, Discrete and Continuous transfers, Introduction to tracer method in Physiology, Bath-tub models, Continuous Infusion into a Compartment, Elementary pharmacokinetics, Parameter estimation in two Compartment models.

Recommended Books:

1. Mathematical Biology (An Introduction, Vol. I and II), J.D. Murray, Springer Verlag.
2. Mathematical Model in Biology and Medicines, J.N. Kapur.
3. S. I. Rubinow, Introduction to Mathematical Biology, John Wiley and Sons, 1975.
4. M. R. Cullen, Linear Models in Biology, Ellis Horwood Ltd.

OPERATIONS RESEARCH

Course No. MM-CP-307 — ***Maximum Marks:100***
Duration of Examination: 3 hrs — ***(a) External Exam: 80***
(b) Internal Exam: 20

Unit I

Unit I

Definition of Operational Research, main phases of OR study, Linear programming problems (LPP), applications to industrial problems –optimal product links and activity levels, convex sets and convex functions, simplex method and extreme point theorems. Big M and two phase methods of solving LPP.

Unit II

Revised simplex method, Assignment problem, Hungarian method, Transportation problem, and Mathematical formulation of transportation problem, methods of solving (North-West Corner rule, Vogel's method and U.V. method.) Concept and applications of duality, formulation of dual problem, duality theorems (weak duality and strong duality theorems), dual simplex method, primal- dual relations, complementary slackness theorems and conditions.

Unit III

Sensitivity Analysis: changes in the coefficients of the objective function and right hand side constants of constraints, adding a new constraint and a new variable. Project management: PERT and CIM: probability of completing a project.

Unit IV

Game theory: Two person zero sum Games, games with pure strategies, Games with mixed strategies, Min. Max. principle, Dominance rule, finding solution of 2 x 2, 2 x m, 2 x m games. Equivalence between game theory and linear programming problem(LPP). Simplex method for game problem. Queues: Empirical models (M/ M/1): (GD/∞/∞) (M/M/C : (GD/∞/∞) model and (M/M/!): (GD/N/ ∞) model.

Recommended Books:

1. Curchman C.W Ackoff R.L and Arnoff E.L (1957) Introduction to Operations Research.
2. F. S Hiller and G.J. Lieberman: Introduction to Operations Research (Sixth Edition), McGraw Hill International, Industries Series, 1995.
3. G. Hadley : Linear programming problem, Narosa publishing House, 1995.
4. Gauss S.I : Linear Programming : Wiley Eastern
5. Kanti Swarup, P.K Gupta and Singh M. M: Operation Research; Sultan Chand & Sons.

Suggested Readings:

1.Philips D.T., Ravindran A. and Solberg J. Operation Research, Principles and Practice.

2.Taha H.A (1982) Operational Research : An Introduction; McMillan.

3.Curchman C.W Ackoff R.L and Arnoff E.L (1957), Introduction to Operations Research.

4.F. S Hiller and G.J. Lieberman: Introduction to Operations Research (SixthEdition) , McGraw Hill International, Industries Series, 1995.

5.G. Hadley : Linear programming problem, Narosa publishing House, 1995.

6.Kanti Swarup, P.K Gupta and Singh M. M: Operation Research; Sultan Chand & Sons.

7.S.S Roa : Optimization Theory and Application, Wiley Eastern Ltd. New Delhi.
8.Taha H.A, Operational Research, An Introduction; Macmillan(1995).

COMPUTER PROGRAMMING

Course No. MM-CP-308 **Maximum Marks: 100**
Duration of Examination: 3 hrs **(a) External Exam: 80**
(b) Internal Exam: 20

Unit I

Introduction to Programming and Problem Solving – The Basic Model of Computation, Algorithms, Flow-charts, Programming Languages, Compilation, Linking and Loading, Testing and Debugging, Documentation.

Introduction to C Language – Character set, Variables and Identifiers, Built-in Data Types, Variable Definition, Arithmetic Operators and Expressions, Constants and Literals, Simple Assignment Statement, Basic Input/Output statements, Simple C Programs.

Conditional Statements and Loops – Decision making with a program, Conditions, Relational Operators, Logical Connectives, *if* statement, *if-else* statement, Loops: *while* loop, *do-while* loop, *for* loop, Nested Loops, Infinite Loops, switch Statement, Structured Programming.

Unit II

Arrays – One Dimensional Arrays: Array Manipulation; Searching, Insertion, Deletion of an element from an Array, Finding the largest/smallest element in an array, Two Dimensional Arrays: Addition/Multiplication of two matrices, Transpose of a square Matrix, Null Terminated Strings as Array of Characters, Representation of Sparse Matrices.

Pointers - Address operators, Pointer type declaration, Pointer assignment, Pointer Initialization, Pointer arithmetic, Function and pointers, Arrays and pointers, Pointer Arrays.

Unit III

Functions - Top Down approach of problem solving, Modular Programming and functions, Standard library of C functions, Prototype of a function, Formal parameter list, return type, Function call, Block Structure, Passing Arguments to a function: call by value; call by reference, Recursive functions, Arrays as function arguments.

Structures and Unions - Structure variables, Initialization, Structure Assignment, nested structure, Structures and functions, Structure and arrays: Arrays of structures, Structures containing arrays, Unions.

File Processing - Concept of files, File opening in various modes and closing of a file, Reading from a file and writing into a file.

Unit IV

Applications of C Language in Mathematics - Implementation of following methods using C programs: Bisection method ,Newton-Raphson Method, Gauss Elimination, Gauss Siedel Method, Iteration Method, Solution of 1st and 2nd Order Differential Equations Using Runga Kutta Method, Picard Method, Euler’s Method and Predictor and Corrector method.

Writing C Programs for Binomial, Trinomial, and Multinomial Distributions.

Recommended Books:

1. Bryon Gottfried, “Programming with C”
2. E. Balaguruswamy, “Programming with ANSI C”
3. A. Kamthane, “Programming with ANSI & Turbo C”

ADVANCED TOPICS IN LINEAR ALGEBRA

Course No. MM-CP-309 ***Maximum Marks: 100***
Duration of Examination: 3 hrs ***(a) External Exam: 80***
(b) Internal Exam: 20

UNIT-I
Tensor product of vector spaces, isomorphism of Hom(V,W) with V* tensor W, tensor algebra, symmetric algebra,

UNIT-II
Exterior algebra of a vector space with their universal properties, structure of bilinear forms, symmetric and alternating forms, orthogonal transformations, reflections.

UNIT-III
Hermitian forms, classical groups associated to Symmetric and Alternating bilinear forms as isometry groups (namely, SO(V,Q), O(V,Q), Sp(V,Q))

UNIT-IV
Spectral theorem, Pfaffian, Witt's Cancellation and Extension theorem for quadratic spaces (without proof). Theorem of Cartan-Dieudonne on orthogonal transformations.

Recommended Books:

1.S.Lang, Introduction to Linear Algebra, Springer Verlag (1987).

SEMESTER: 4

PARTIAL DIFFERENTIAL EQUATIONS

Course No. MM-CP-401 ***Maximum Marks: 100***
Duration of Examination: 3 hrs ***(a) External Exam: 80***
(b) Internal Exam: 20

UNIT I
Partial Differential Equations of first order PDEs, origins of first order PDEs, Cauchy Problem for first order equations, Linear. equations of the first order, Nonlinear PDEs of the first order, Lagrange and Charpits methods for solving first order PDEs.
Unit II

Classification of Second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Unit III

Three Basic Equations, the Wave equation---one dimensional case, D'Alembert's solution, the initial value problem in three space, Poisson's method of spherical averages, Hadamard's method of descent, Duhamel's Principle, the inhomogeneous wave equation.

Unit IV

Fourier transform, definition, Fourier transform---L^1-theory, Rieman-Lebesgue theorem, L^2-theory, Plancheral theorem, L^p-theory, Cauchy-Kowalewska theorem.

Recommended Books

Partial Differential Equations by Fritz John, Springer Verlag
Partial Differential Equations by Ian Sneddon, McGraw Hill
Partial Differential Equations by L.C. Evans, GTM, AMS, 1998
Partial Differential Equations by P. Prasad and R. Ravindaran,
Partial Differential Equations by Amarnath.

DIFFERENTIAL GEOMETRY

Course No. MM-CP-402 ***Maximum Marks: 100***
Duration of Examination: 3 hrs ***(a) External Exam: 80***
(b) Internal Exam: 20

Unit I

Curves : Differentiable curves, Regular point, Parameterization of curves, arc-length, and arc-length is independent of parameterization, unit speed curves. Plane curves: Curvature of plane curves, osculating circle, centre of curvature. Computation of curvature of plane curves. Directed curvature, fundamental theorem for plane curves. Examples: Straight line, circle, ellipse, tractrix, evolutes and involutes. Space curves: Tangent vector, unit normal vector and unit binormal vector to a space curve.

Curvature and torsion of a space curve. The Frenet-Serret theorem. First Fundamental theorem of space curves. Intrinsic equation of a curve. Computation of curvature and torsion. Characterization of Helices and curves on sphere in terms of their curvature and torsion. Evolutes and involutes of space curves**.**

Unit II

Surfaces; Regular surfaces with examples, coordinate charts or curvilinear coordinates, change of coordinates, tangent plane at a regular point, normal to the surface, orient able surface, differentiable mapping between regular surfaces and their differential. Fundamental form or a metric of a surface, line element, invariance of a line element under change of coordinates, angle between two curves, condition of orthogonality of coordinate curves. Area of bounded region, invariance of area under change of coordinates.

Unit III

Curvature of a Surface: Normal curvature, Euler's work on principal curvature,. Qualitative behavior of a surface near a point with prescribed principal curvatures. The Gauss map and its differential. The differential of Gauss is self-adjoint. Second fundamental form. Normal curvature interms of second fundamental form. Meunier theorem. Gaussian curvature, Weingarten equation. Gaussian curvature $K(p)= (eg-f^2)/EG-F^2$ .surface of revolution. Surfaces with constant positive or negative Gaussian curvature. Gaussian curvature in terms of area. Line of curvature, Rodrigue's formula for line of curvature, Equivalence of Surfaces: Isometry between surfaces, local isometry, and characterization of local isometry.

Unit IV.

Christoffel symbols. Expressing Christoffel symbols in terms of metric coefficients and their derivative. Theorema egregium (Gaussian curvature is intrinsic). Isometric surfaces have same Gaussian curvatures at corresponding points. Gauss equations and Manardi Codazzi equations for surfaces. Fundamental theorem for regular surface. (Statement only).

Geodesics: Geodesic curvature, Geodesic curvature is intrinsic, Equations of Geodesic, Geodesic on sphere and pseudo sphere. Geodesic as distance minimizing curves. Gauss-Bonnet theorem (statement only).Geodesic triangle on sphere. Implication of Gauss-Bonnet theorem for Geodesic triangle.

Recommended Books:

1. John Mc Cleary: Geometry from a differentiable Viewpoint. (Cambridge Univ. Press)

Suggested Readings:

1. W. Klingenberg: A course in Differential Geometry (Spring Verlag)
2. C.E. Weatherburn: Differential Geometry of Three dimensions.
3. T. Willmore : An Introduction to Differential Geometry
4. J. M. Lee : Riemannian Manifolds, An Introduction to Curvature (Spring)

ADVANCED TOPICS IN FUNCTIONAL ANALYSIS

Course No. MM-CP-403 ***Maximum Marks: 100***
Duration of Examination: 3 hrs ***(a) External Exam: 80***
(b) Internal Exam: 20

Unit I

Topological vector spaces (TVC), Definition and Examples. Basic properties – subspaces quotients and products of TVS, Bounded sets & totally bounded sets. characterizing a linear topology in terms of a local' base. Continuous and bounded linear maps between TVS.

Unit II

Projective and inductive topologies, Projective and inductive limits of linear topologies. Weak topology of a TVS, Metrization in TVS. Completeness, sequential completeness and quasi completeness in TVS and their relationship. F-spaces and open mapping theorem/closed-graph theorem in F-spaces.

Unit III

Locally convex spaces & their characterizations. Hahn-Banach theorem & its simple consequences. Duality & polar topologies. Compatible linear (locally convex) topologies, Mackey-Arens Theorem.

Unit IV

Duality invariance of bounded & closed convex sets. Equicontinuity and Alaoglu-Bourbaki theorem. Bipolar theorem. Barrelled infrabarrelled and bornological spaces. Banach-Steinhauss theorem.

Recommended Books

1 Wilansky, A: Modern Methods in Toplogical Vector Spaces(McGraw Hill).
2 Swartz, C: Topological Vector Spaces (Marcel Dekker).

Suggested Readings:

1. Rudin, W: Functional Analysis (Tata McGrawHill).
2. Jarchow ,H,: Locally Convex Spaces (Teubner Texts).
3. Schaefer, H,H.: Topological Vector Spaces (Springer Verlag).
4. Bachman, G & Narici, L: Topological Vector Spaces (Marcel Dekker).

ADVANCED TOPICS IN THE ANALYTICAL THEORY OF POLYNOMIALS

Course No. MM-CP-404	***Maximum Marks: 100***
Duration of Examination: 3 hrs	***(a) External Exam: 80***
	(b) Internal Exam: 20

Unit I

The fundamental theorem of algebra (revisited), symmetric polynomials, The Continuity theorem, Orthogonal Polynomials, General Properties, The Classical Orthogonal Polynomials, Harmonic and Sub Harmonic functions, Tools from Matrix Analysis.

Unit II

Critical points in terms of zeros, Fundamental results on critical points, Convex Hulls and Gauss-Lucas theorem, Some applications of Gauss-Lucas theorem. Extensions of

Gauss-Lucas theorem, Average distance from a line or a point Real polynomials and Jenson's theorem , Extensions of Jenson's theorem.

Unit III

Derivative estimates on the unit disc, Bernstein's inequality and generalizations. Refinements, Conditions on the coefficients, Inequalities for polynomials having all zeros on the unit circle. Self-reciprocal polynomials, conditions on the zeros. Inequalities for polynomials involving mean values.

Unit IV

Inequalities of S. Bernstein and A. Markov on the unit interval, Extensions of higher order derivatives. Estimates for individual coefficients of polynomials, Inequalities involving two coefficients, Inequalities involving all the coefficients, Coefficient estimates of real trigonometric polynomials. Sharp estimates for individual coefficients.

Recommended Books

1. Analytic theory of Polynomials by Q.I. Rahman and G.Schmeisser.
2. Geometry of polynomials by Morris Marden.

Suggested Readings:

1. Topics in polynomials :extremal properties, problems, inequalities, zeroes by G.V.Milovanovic,D.S.Mitrinovic and Th. M. Rassias
2. Problems and theorems in Analysis II by G.Polya and G.Szego (Springer Verlag New York Heidelberg Berlin).

THEORY OF NUMBERS-II

Course No. MM-CP-405
Duration of Examination: 3 hrs

Maximum Marks: 100
(a) External Exam: 80
(b) Internal Exam: 20

Unit-I

Integers belonging to a given exponent mod p and related results. Converse of Fermat's Theorem. If $d/p-1$, the Congruence $x^d \equiv 1 (\text{mod } p)$, has exactly d-solutions. If any integer belongs to t (mod p), then exactly $\phi(t)$ incongruent numbers belong to t(mod p). Primitive roots. There are ϕ (p-1) primitive roots of a odd prime p. Any power of an odd prime has a primitive root. The function λ (m) and its properties. $a^{\lambda(m)} \equiv 1 \pmod{m}$, where (a , m)=1.There is always an integer which belongs to λ(m) (mod m). Primitive λ-roots of m.The numbers having primitive roots are 1,2, 4, p^α and $2p^\alpha$.where p is an odd prime.

Unit II

Quadratic residues. Euler criterion. The Legendre symbol and its properties. Lemma of Gauss. If p is an odd prime and (a, 2p) =1,

then $\left(\frac{a}{p}\right)=(-1)^t$ where $t=\sum_{j=1}^{(p-1)/2}\left[\frac{ja}{p}\right]$ and $\left(\frac{2}{p}\right)=(-1)^{(p^2-1)/8}$

The law of a Quadratic Reciprocity, Characterization of primes of which 2,-2, 3,-3, 5, 6 and 10 are quadratic residues or non residues. Jacobi symbol and its properties. The reciprocity law for Jacobi symbol.

Unit III

Number theoretic functions. Some simple properties of $\tau(n)$,$\sigma(n)$, $\phi(n)$ and $\mu(n)$. Mobius inversion formula. Perfect numbers. Necessary and sufficient condition for an even number to be perfect. The function [x] and its properties. The symbols "O", "o", and " ~" .Euler's constant γ . The series $\sum_p 1/p$ diverges. $\prod_{p\le n} p<4^n$, for $n\geq 2$. Average order of magnitudes of $\tau(n)$, $\sigma(n)$, $\phi(n)$. Farey fractions. Rational approximation.

Unit IV

Simple continued fractions. Application of the theory of infinite continued fractions to the approximation of irrationals by rationals. Hurwitz theorem. $\sqrt{5}$ is the best possible constant in the Hurwitz theorem. Relation between Riemann Zeta function and the set of primes. Characters. The L-Function L(S, χ) and its properties. Dirichlet's theorem on infinity of primes in an arithmetic progression (its scope as in Leveque's topics in Number Theory, Vol. II. Chapter VI).

Recommended Books

1. Topics in number theory by W. J. Leveque, Vol. I and II Addition Wesley Publishing Company, INC.

2. An introduction of the Theory of numbers by I. Niven and H.S Zucherman.

3. Number Theory by Boevich and Shafeviech, I.R Academic Press.

<u>Suggested Readings:</u>

1. Analytic Number Theory by T.M Apostal, Springer international.

2. An introduction to the theory of Numbers by G.H Hardy and Wright.

3. A course in Arithmetic, by J.P. Serre, GTM Vol. Springer Verlag 1973.

4. An elementary Number theory of E. Landau.

<u>ADVANCED TOPICS IN DISCRETE MATHEMATICS</u>

Course No. MM-CP-406 — ***Maximum Marks: 100***
Duration of Examination: 3 hrs — ***(a) External Exam: 80***
(b) Internal Exam: 20

Unit-I

Graph matrices, Incidence matrix A(G), Isomorphic of graphs in terms of their incidence matrices, Rank of incidence matrix, the rank of incidence matrix of connected graph with n vertices, sub matrices of A(G), Cur-set matrix C(G), Rank C(G)=rank of A(G)= rank of G, relationships between A_j , B_j , C_j ,Path matrix, Adjacency matrix X(G), Powers of X, Relation between A(G) and X(G).

Unit II
Coloring, chromatic number $\chi(G)$, A graph is bicolorable iff it has no odd cycles, Bounds for $\chi(G)$, Bounds on sum and product of the chromatic number of a graph and its complement, Five color theorem, Four color theorem (statement only), Every planar graph is four colorable iff every cubic bridgeless plane map is 4-colorable, Every planar graph is 4-colorable iff $\chi'(G)=3$ for every bridgeless planar graph, Heawood Map-coloring theorem, Uniquely colorable graphs.

Unit III
Edge graphs, A connected graph is isomorphic to its edge graph iff it is a cycle, Whitney's theorem on edge graphs, Characterization of edge graphs of trees, edge graphs and traversibility, total graphs, Eccentricity sequence and sets, Lesniak theorem for trees, Construction of trees, Neighborhoods, Lesniak theorem for graphs.

Unit IV
Digraphs, types of digraphs, Digraphs and binary relations, Directed paths and connectedness, Euler digraphs, Trees with directed edges, Arborescence, Ordered trees, Spanning arborescence, Fundamental cycles in digraphs, Matrices A, B, C of digraphs, The determinant of every square sub matrix of A is 1, -1 or 0. Rows of cycle matrix are orthogonal to the rows of the incidence matrix, Number of spanning trees, Fundamental cycle matrix, Adjacency matrix of a digraph, Connectedness and the adjacency matrix, Number of arborescence, tournaments, score sequences, Landau's theorem, Oriented graphs.

Recommended Books

1. F. Harary , Graph Theory, Addison- Wesley.
2. F. Harary, F.R. z. Norman and D. Cartwright ; Structure Models : An Introduction to the theory of Directed graphs, J. Wiley.

Suggested Readings:

1. K.R Parthasarty : Basic Graph Theory, Tata Mc-Graw Hill
2. B. Bollobas : External Graphs theory, Acad, Press London.
3. D.B. West Introduction to Graph Theory, Prentice Hall.
4. T.L.Saaty and P.C. Kainen : The Four Color Problem, Dover Pub.
5. S.Pirzada and A.Dharwadkar, Graph Theory, Universities Press(Orient Longman)

MATHEMATICAL STATISTICS

Course No. MM-CP-407 ***Maximum Marks: 100***
Duration of Examination: 3 hrs ***(a) External Exam: 80***
(b) Internal Exam: 20

Unit I

Some Special Distributions, Bernoulli, Binomial, Trinomial, Multinomial, Negative Binomial, Poisson, Gamma, Chi-square, Beta, Cauchy, Exponential, Geometric, Normal and Bivariate Normal Distributions.

Unit II

Distribution of Functions of Random Variables, Distribution Function Method, Change of Variables Method, Moment generating function Method, t and F Distributions,

Dirichelet Distribution, Distribution of Order Statistics, Distribution of X and $\frac{nS^2}{\sigma^2}$,

Limiting distributions, Different modes of convergence, Central Limit theorem.

Unit III

Interval Estimation, Confidence Interval for mean, Confidence Interval for Variance, Confidence Interval for difference of means and Confidence interval for the ratio of variances. Point Estimation, Sufficient Statistics, Fisher-Neyman criterion, Factorization Theorem, Rao- Blackwell Theorem, Best Statistic (MvUE), Complete Sufficient Statistic, Exponential class of pdfs.

Unit IV

Rao-Crammer Inequality, Efficient and Consistent Estimators, Maximum Likelihood Estimators (MLE's).

Testing of Hypotheses, Definitions and examples, Best or Most powerful (MP) tests, Neyman Pearson theorem, Uniformly most powerful (UMP) Tests, Likelihood Ratio Test, Chi-square Test.

Recommended Books

1 Hogg and Craig : An Introduction to Mathematical Statistics
2 Mood and Grayball : An Introduction to Mathematical Statistics

Suggested Readings:

1. C.R.Rao : Linear Statistical Inference and its Applications
2. V.K.Rohatgi : An Introduction to Probability and Statistics.

WAVELET ANALYSIS

Course No. MM-CP-408 — ***Maximum Marks: 100***
Duration of Examination: 3 hrs — ***(a) External Exam: 80***
(b) Internal Exam: 20

Unit-I: ELEMENTS OF FOURIER ANALYSIS: Fourier series, Fourier transforms, Inversion formula, Parseval Identity and Plancherel Theorem, Continuous-time convolution and the delta function, Heisenberg uncertainty principle , Poisson's summable formula, Shannon sampling theorem, Fourier transforms of tempered distributions

Unit-II: WAVELET TRANSFORM: Time - frequency localization, definition and examples of wavelets, Dyadic wavelets, Wavelet series, Orthonormal wavelet bases, continuous and discrete wavelet transform, frames.

Unit-III: SCALING FUNCTIONS AND MULTI-RESOLUTION ANALYSIS (MRA): Multiresolution analysis, orthonormal systems and Riesz systems, scaling equations and structure constants, from scaling function to MRA and orthonormal wavelet.

Unit –IV: COMPACTLY SUPPORTED WAVELETS AND CONVERGENCE PROPERTIES: Spline wavelets and their properties, wavelets with compact support, construction of compact wavelets, smoothness of wavelets, convergence properties of wavelet series.

Recommended Books

1. Ten lectures of wavelets, Daubechies, I, CBMS series, Philadelphia, SIAM, 1992.
2. Introduction to Fourier analysis and wavelets, Pinsky, M, A, Brooks/Cole 2002.

1. A first course on wavelets, Hernandez, E and G. Weiss, Boca Raton, FL, CRC press, 1996.
2. An introduction to wavelets, Chui, C.K, San Diego, Academic press, 1992

BANACH ALGEBRAS AND SPECTRAL THEORY

Course No. MM-CP-409	***Maximum Marks: 100***
Duration of Examination: 3 hrs	***(a) External Exam: 80***
	(b) Internal Exam: 20

Unit-I

Banach Algebra:- Preliminaries on Banach Algebra's Invertible elements, the spectrum, spectral radius and the spectral radius formula, Gelfand- Mazur theorem, Gelfand mapping, maximal ideal space and its characterization, continuity of multiplicative functionals on a Banach algebra.

Unit-II

B* -Algebra and the Gelfand Naimark Theorem, Ideals in C(X) and application to stone-Cech conpactification and Banach stone theorem, structure of commutative C* -Algebras.

Unit-III

Compact operators in Banach spaces, spectral theorem for compact Hermitian operators, spectral theorem for compact normal operators and its consequences.

UNIT-IV

Invariant subspace problem and its validity for compact Hermitian operators, Lomonosov's theorem on the existence of invariant subspaces for operators commuting with compact operators.

Recommended Books

1.J.B. Conway, A course in Functional Analysis (GTM 96, Springer Verlag**).**
2.K.Saxe, Beginning Functional Analysis, Springer Verlag.
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